

Lesson 1.2

Proportions as Models

As you saw in the previous lesson, proportions can be used as mathematical models to help estimate animal populations. In this lesson, you will explore how proportions can be used to model a variety of other real-world situations.

WRITING AND SOLVING PROPORTIONS

When you write a proportion to represent a given situation, be sure that the quantities in each ratio are written in the same order. For example, you know that there are 12 inches in 1 foot and there are 36 inches in 3 feet. You can write a proportion to model how these quantities are related.

$$\begin{array}{ccccccc} \text{inches} & \rightarrow & 12 \text{ inches} & = & 36 \text{ inches} & \leftarrow & \text{inches} \\ \text{feet} & \rightarrow & 1 \text{ foot} & = & 3 \text{ feet} & \leftarrow & \text{feet} \end{array}$$

Notice that because the ratio on the left is expressed as “inches to feet,” the ratio on the right must also be expressed as “inches to feet.”

EXAMPLE 1

According to the American Automobile Association (AAA), the overall cost of owning and operating a passenger vehicle averages \$7,834 based on 15,000 miles of driving. If the cost per mile is constant, about what would it cost to drive 12,000 miles?

Solution:

Let c represent the cost of driving 12,000 miles.

Write a proportion for the problem.

$$\begin{array}{ccccccc} \text{average cost} & \rightarrow & \$7,834 & = & c & \leftarrow & \text{average cost} \\ \text{number of miles} & \rightarrow & 15,000 & = & 12,000 & \leftarrow & \text{number of miles} \end{array}$$

Solve for c .

$$\text{Original equation} \quad \frac{7,834}{15,000} = \frac{c}{12,000}$$

$$\text{Find the cross products.} \quad 15,000c = (7,834)(12,000)$$

$$\text{Simplify.} \quad 15,000c = 94,008,000$$

$$\text{Divide each side by 15,000.} \quad \frac{15,000c}{15,000} = \frac{94,008,000}{15,000}$$

$$\text{Simplify.} \quad c = 6,267.20$$

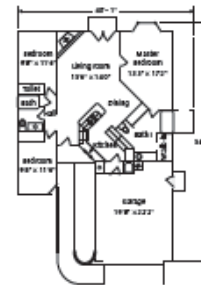
So, the average cost of driving 12,000 miles is about \$6,267.

SCALE DRAWINGS

Scale drawings are used in many types of design work to accurately model the shapes of objects. A scale is a ratio that compares the size of a model to the actual size of an object. Scales are often found on drawings, maps, and models.

EXAMPLE 2

A typical scale for a house plan is $\frac{1}{4}$ inch to 1 foot. If the width of a room on such a plan measures $3\frac{1}{2}$ inches, what is the actual width of the room?



Solution:

Let w represent the actual width of the room.

Write a proportion to model the situation.

$$\begin{array}{ccccccc} \text{drawing (in.)} & \rightarrow & \frac{1}{4} & = & \frac{3\frac{1}{2}}{w} & \leftarrow & \text{drawing (in.)} \\ \text{actual room (ft)} & \rightarrow & 1 & = & w & \leftarrow & \text{actual room (ft)} \end{array}$$

Solve for w .

$$\text{Original equation} \quad \frac{1}{4} = \frac{3\frac{1}{2}}{w}$$

$$\text{Find the cross products.} \quad \frac{1}{4}w = 3\frac{1}{2}(1)$$

$$\text{Multiply each side by 4.} \quad (4)\frac{1}{4}w = (4)\left(3\frac{1}{2}\right)$$

$$\text{Simplify.} \quad w = 14$$

So, the width of the room is 14 feet.

Recall

A polygon is a closed plane figure formed by line segments called sides that meet only at their endpoints. Each point where the sides meet is called a vertex.

SIMILAR POLYGONS

Two figures that have the same shape, but not necessarily the same size, are said to be similar.

Two polygons are similar polygons if their corresponding angles are equal in measure and the lengths of their corresponding sides are proportional.

Recall

Congruent figures have the same size and shape.

It is also the case that if two polygons are similar, then you know that the corresponding angles are congruent and the corresponding sides are proportional.

If two polygons are similar, the ratio of the lengths of two corresponding sides is called the scale factor.

EXAMPLE 3

Given: $ABCD \sim KLMN$



- What is the scale factor of $ABCD$ to $KLMN$?
- Find the value of x .

Solution:

- \overline{AB} and \overline{KL} are corresponding sides of the two quadrilaterals.

So, the scale factor is $\frac{AB}{KL} = \frac{4}{6} = \frac{2}{3}$.

- Since the polygons are similar, you know the following:

$\angle A \cong \angle K$, $\angle B \cong \angle L$, $\angle C \cong \angle M$, and $\angle D \cong \angle N$.

Also, $\frac{AB}{KL} = \frac{BC}{LM} = \frac{CD}{MN} = \frac{DA}{NK}$.

To find the value of x , write a proportion and solve.

Corresponding sides of similar polygons are proportional. $\frac{AB}{KL} = \frac{BC}{LM}$

$$AB = 4, BC = 5, KL = 6, LM = x$$

Find the cross products.

Simplify.

Divide each side by 4.

Simplify.

$$\frac{4}{6} = \frac{5}{x}$$

$$4x = (5)(6)$$

$$4x = 30$$

$$\frac{4x}{4} = \frac{30}{4}$$

$$x = 7.5$$

Recall

When similar polygons are named, the corresponding vertices are listed in the same order.

Practice for Lesson 1.2

For Exercises 1–3, choose the correct answer.

- Which proportion *cannot* be used to solve the following problem?

How many milligrams (mg) of medication should you give to a 120-pound person if you should give 50 mg for every 10 pounds?

A. $\frac{50 \text{ mg}}{10 \text{ lb}} = \frac{x}{120 \text{ lb}}$

B. $\frac{10 \text{ lb}}{50 \text{ mg}} = \frac{120 \text{ lb}}{x}$

C. $\frac{50 \text{ mg}}{x} = \frac{120 \text{ lb}}{10 \text{ lb}}$

D. $\frac{10 \text{ lb}}{120 \text{ lb}} = \frac{50 \text{ mg}}{x}$

- Triangles ABC and XYZ are similar. Which statement is *not* true?

A. $\frac{AB}{XY} = \frac{BC}{YZ}$

B. $\frac{XZ}{AC} = \frac{YZ}{BC}$

C. $\frac{CB}{ZY} = \frac{AC}{XZ}$

D. $\frac{XY}{AB} = \frac{ZY}{CA}$

- $ABCD$ is a rectangle.



Which set of dimensions produces a rectangle that is similar to rectangle $ABCD$?

A. 36.4 mm, 11 mm

B. 44 mm, 9.1 mm

C. 176 mm, 72.8 mm

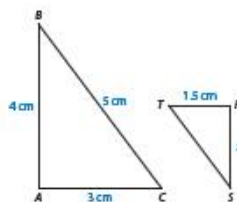
D. 91 mm, 66 mm

- If your new car goes 320 miles on 10 gallons of gas, how far will it go on 6 gallons of gas?
- The Tannery Mall in Massachusetts is partially powered by an array of 375 solar panels. They produce 60 kilowatts of electrical power. How many panels would be needed to produce 84 kilowatts of power?

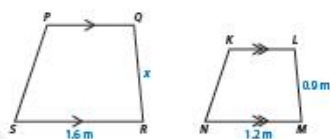


6. An airplane sprays 16 gallons of liquid fertilizer on 5 acres of crops. If the plane's tank can hold 280 gallons, how many acres of crops can be sprayed?
7. Most conventional TV screens have a width : height ratio of 4 : 3. If a screen has a width of 42 inches, what is its height?
8. The scale on a map is 1 inch : 6 miles. Find the actual length of a road if it is 3 inches long on the map.
9. A drawing's scale is 0.5 inch : 20 feet. If a banquet room's length is 50 feet, what is the length of the room in the drawing?

10. Given: $\triangle ABC \sim \triangle RST$



- a. What is the scale factor of triangle ABC to triangle RST ?
 - b. Find the value of x .
11. Trapezoid $PQRS$ is similar to trapezoid $KLMN$. Find the value of x .



12. The ratio of the corresponding sides of two similar rectangles is 4 : 9. The length of the smaller rectangle is 16 cm and its width is 12 cm. What is the perimeter of the larger rectangle?
13. Suppose that a , b , c , and d represent four numbers that form the proportion $\frac{a}{b} = \frac{c}{d}$. If a is doubled while b remains the same, how would c or d have to change for the proportion to stay true?