Suppose that the pump manufacturer referred to in the Chapter Opener on page 492 has such poor quality control that one-fourth of its pumps are defective. If each airplane has two pumps, you can use what you know about probability to determine the probability that both of these pumps are defective, exactly one pump is defective, or both pumps are nondefective.

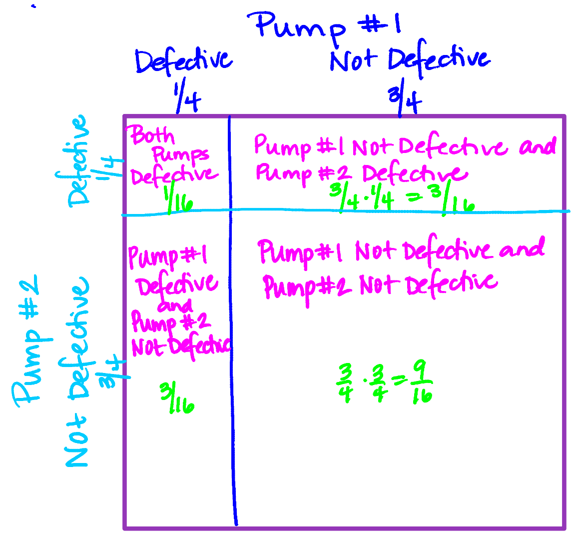
1. Draw a square to represent a total probability of 1 or 100%. Probabilities for all possible pump combinations will be represented in the square. Draw a vertical line that divides the square into two rectangles. One rectangle should contain $\frac{1}{4}$ of the total area. These rectangles represent the probabilities that Pump #1 is either defective or nondefective.

   Pump #1

   Write appropriate labels along the top side of the square. Be sure to include the probability values on your area models.

2. Now draw a horizontal line that divides the left side of the square in a similar way. Write labels along the left side to show the probabilities that Pump #2 is either defective or nondefective.

3. The square is now divided into four regions. The smallest region represents the event that both pumps are defective. Label each region according to the combination of pump conditions it represents.



4. Complete the table below by computing the areas of the regions to find the probability of each state of the system.

State of the System	Probability
Both pumps defective	1/16
Pump #1 defective, Pump #2 nondefective	3/16
Pump #1 nondefective, Pump #2 defective	3/16
Both pumps nondefective	9/16

Joint Probabilities: A combination of probabilities of 2 events both occurring

5. The probabilities in the table are called *joint probabilities* since each of them is a combination of the probabilities of two events. Whether the system works or not depends on two separate events *both* occurring. Examine how you computed each probability. Then write a rule for finding the probability that two events both occur.

To find the probability that two events both occur, you have to multiply the probabilities of the individual events.

6. Suppose an airline has 80 aircraft of this type. Use your answer to Question 4 to help you complete the table below. Fill in the expected number of airplanes in each category.

		Fuel Pump #1	
		Defective	Nondefective
Fuel Pump #2	Defective	$\frac{1}{16} \cdot 80 = 5$	$\frac{3}{16} \cdot 80 = 15$
	Nondefective	$\frac{3}{16} \cdot 80 = 15$	$\frac{9}{16} \cdot 80 = 45$

7. How many of the 80 planes have exactly 1 defective fuel pump? $15 + 15 = 30$
8. What is the probability that a plane has exactly 1 defective pump? $30/80 = 3/8$
9. Explain how you could use your answer to Question 4 to find the probability that a plane has exactly 1 defective pump.

$$\underbrace{\frac{3}{16} + \frac{3}{16}}_{} = \frac{6}{16} = \frac{3}{8}$$

Add the probabilities for the 2 situations that have 1 defective pump and 1 nondefective pump.

INDEPENDENT AND DEPENDENT EVENTS

In the Investigation, you found that the joint probability that both fuel pumps fail could be found by multiplying the failure probabilities for the individual pumps. This assumes that fuel pump failures are **independent** events. That is, the occurrence of one event does not depend on the occurrence of the other event. In this case, the probability that one of the pumps fails does not depend on whether the other pump fails.

Probability of Independent Events

If two events, A and B, are independent, then the probability that both occur is the product of the probability of A and the probability of B.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

What if the events are not independent? That is, what if the failure of Pump #1 made Pump #2 more (or less) likely to fail? In this case, the events are **dependent**. That is, the occurrence of one event affects the occurrence of the other. In such a case, a conditional probability would have to be used for the failure probability of Pump #2. And the probability that both pumps fail would be found from

$$P(\text{both pumps fail}) = P(\text{Pump \#1 fails}) \cdot P(\text{Pump \#2 fails} \mid \text{Pump \#1 fails}).$$

Probability of Dependent Events

If two events, A and B, are dependent, then the probability that both occur is the product of the probability of A and the probability of B given that A also occurs.

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

Independent -vs- Dependent.

Independent	Dependent
1 event does not affect the outcome of the other event	1 event DOES affect the probability of the next event

1. Draw 2 cards:

52 cards
4 suits
13 of each card

a) P(2 ♡'s) w/replacement $= \frac{13}{52} \cdot \frac{13}{52} = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$ *Independent*

b) P(2 ♡'s) no replacement $= \frac{13}{52} \cdot \frac{12}{51} = \frac{1}{\cancel{4}} \cdot \frac{\cancel{4}}{17} = \frac{1}{17}$ *Dependent*

c) P(♡, then ◇) no replacement $= \frac{13}{52} \cdot \frac{13}{51} = \frac{1}{4} \cdot \frac{13}{51} = \frac{13}{204}$ *Dependent*

2. Pg 506

According to the Asthma and Allergy Foundation of America, one-fifth of all Americans have allergies of some kind. But if a person has one parent with allergies, that person has a one-third chance of having allergies.

a. What is the probability that two randomly selected Americans both have allergies?

b. What is the probability that four randomly selected Americans all have allergies?

c. What is the probability that a mother and her child both have allergies?

a) P(2 random Am. have allergies) $= \frac{1}{5} \cdot \frac{1}{5} = \frac{1}{25}$

b) P(4 " ") $= \frac{1}{5} \frac{1}{5} \frac{1}{5} \frac{1}{5}$ or $\left(\frac{1}{5}\right)^4 = \frac{1^4}{5^4} = \frac{1}{625}$

c) P(adult and child) $= \frac{1}{5} \cdot \frac{1}{3} = \frac{1}{15}$