10.5 Activity Key

You have seen that a type of increase called *exponential growth* and a type of decrease called *exponential decay* involve variable exponents. You have also studied the Properties of Exponents. In this lesson, you will examine the properties of exponential growth functions in more detail.

As you saw in the Activity in Lesson 10.1, the number of *layers* of paper doubles with each cut. The total number of sheets after each cut can be modeled by an exponential function of the form $y = a \cdot b^x$, which in this case is $L = 2^n$, where L is the number of layers and n is the number of cuts. In this Investigation, you will focus on the *thickness* of the paper.

1. A *ream* of paper contains 500 sheets. It is almost 2 inches thick. About how thick is one sheet of paper?

$\frac{\text{inches}}{\text{sheet}}$ $\quad \frac{2\text{in}}{500 \text{ sheets}} = \frac{1}{250} = .004$ inches/sheet

2. Complete the table below for the process you used in the Activity in Lesson 10.1, which involved cutting and stacking paper.

Number of Cuts	Number of Layers	Total Thickness of Paper (in.)	Change in Thicknesses (in.)
0	1	0.004	—
1	2	0.008	0.008 − 0.004 = 0.004
2	4	0.016	.016 − .008 = .008
3	8	0.032	.016
4	16	0.064	.032
5	32	0.128	.064
6	64	0.256	.128

3. Consider the relationship between the total thickness and the number of cuts. Is this a linear relationship? Explain.

No, this is not a linear relationship, because there is not a constant rate of change.

4. Using the same data as in Question 3, calculate the *ratios* of successive thicknesses and complete the table below.

Number of Cuts	Number of Layers	Total Thickness of Paper (in.)	Ratios of Successive Thicknesses
0	1	0.004	—
1	2	0.008	$\frac{0.008}{0.004} = 2$
2	4	0.016	$\frac{.016}{.008} = 2$
3	8	0.032	= 2
4	16	0.064	= 2
5	32	0.128	= 2

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4	16	0.064	=2
5	32	0.128	=2
6	64	0.256	=2

5. Consider the relationship between the total thickness and the number of cuts. Is this an exponential relationship? Explain.

6. Write an equation that describes total thickness T as a function of the number of cuts n.

7. What is the initial value of your function? What is the base?

8. How is this function similar to the function $L = 2^n$ that described the number of layers of paper in the Activity in Lesson 10.1? How is it different?

9. If you could cut the paper 20 times, how thick would the stack be?

10. Use a graphing calculator to graph your function. Use a window of $[-3, 3] \times [0, 0.032]$.

11. Can the value of your function ever be 0?

12. What are the problem domain and range? Explain.

13. Change the window on your graph, as needed, to answer this question. How many cuts would be needed to produce a paper stack at least 6 inches thick?

⑤ Yes, it IS an exponential relationship, because the ratio of successive thicknesses are constant.

⑥ $y = a \cdot b^x$ $\qquad T = .004(2)^n$

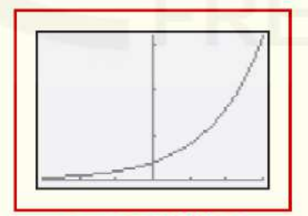
$\uparrow$ initial value $\quad \nwarrow$ successive quotient

⑦ initial value = .004 base = 2

⑧ Similar: same base of 2
difference: initial value of 1 vs. .004
$\uparrow$ # of layers $\quad \uparrow$ thickness

⑨ $T = .004(2)^{20} \approx 4{,}194$ in
(almost 350 ft!!!)

⑩



[−3, 3] × [0, 0.032]

⑪ No

⑫ The problem domain is all whole #'s and the range is {.004, .008, .016, ...} which means the range is $.004(2)^n$ where n is a whole #.

⑬ 11 cuts

14. A mosquito control scientist collected a large sample of mosquitoes each month for five months. The number of mosquitoes in each sample that were infected with West Nile virus were counted.

Month	Month Number	Number of Mosquitoes Testing Positive for West Nile Virus	Successive Ratios
May	0	400	—
June	1	600	$\frac{600}{400} = 3/2$
July	2	900	$\frac{900}{600} = 3/2$
August	3	1,350	3/2
September	4	2,025	3/2

Complete the last column of the table and describe what you find.

15) The successive ratios are all 3/2 or 1.5.

15. Write an equation that models the number of infected mosquitoes M as a function of the month number n.

16. Use a graphing calculator to graph your function for the months of May through September.

17. Describe how you can use a table of values such as the one in Question 14 to determine the equation of an exponential growth function $y = a \cdot b^x$.

$M = 400\left(\frac{3}{2}\right)^n$ or $M = 400(1.5)^n$

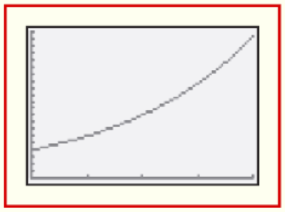
16)

Sample graph:

17. Describe how you can use a table of values such as the one in Question 14 to determine the equation of an exponential growth function $y = a \cdot b^x$.

Recall that the base b of an exponential function can be any positive number except 1. Only numbers greater than 1 produce exponential growth. But as you saw in the Investigation, the base does not have to be an integer.

The constant ratio of successive values for an exponential function is equal to the base b only if the values of the independent variable increase by 1 in successive rows of the table.



[0, 4] × [0, 2100]

17) I could use a table to find, a, the initial value by finding the y-value (output) when x (the input) is zero. I could find the constant multiplier b finding the successive quotients.