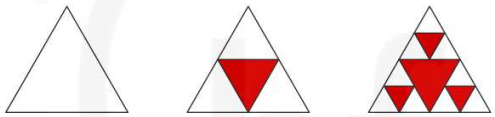
10.3 Day 1 Activity

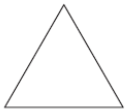
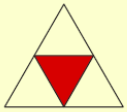
1. A geometric design can be created by repeating a process over and over. The three triangles below show such a process.

   Notice that the second figure is created by connecting the midpoints of the sides of the first unshaded triangle, forming four smaller congruent triangles. Then the central triangle of those four smaller triangles is shaded red.

   The third figure is formed by repeating the process of dividing each of the three unshaded triangles in the second figure into four smaller congruent triangles and then shading the central triangles red.



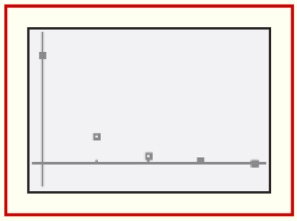
   Assume that this pattern could be continued indefinitely. If the area of the first triangle is exactly 1 square inch, complete the third column of a table like the one below.

Term Number	Picture	Area of the Smallest Triangle (in^2)
0		1
1		$\frac{1}{4}$
2		? $\frac{1}{16}$
3		? $\frac{1}{64}$
4		? $\frac{1}{256}$

2. Use a graphing calculator to make a scatter plot of the area of the smallest triangle vs the term number as the term number increases from 0 to 4. Describe the scatter plot.
3. Write an equation that expresses the area of the smallest triangle A as a function of the term number n.
4. Graph your function on your scatter plot. Does your function fit the scatterplot? If not, revise your function.

$A = \left(\frac{1}{4}\right)^n$ or $A = \frac{1}{4^n}$

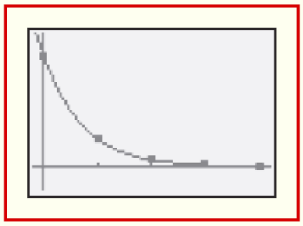
2)



[−0.2, 4.2] × [−0.2, 1.2]

Sample answers:
- The points get closer and closer to the "n-axis."
- The heights of the points decrease, but by smaller and smaller increments.

4)



[−0.2, 4.2] × [−0.2, 1.2]

YES! ☺ The function fits the points in the scatterplot.

5. Use exponent properties to write your function in two different forms. One form should have a positive exponent n. The other should have a negative exponent $-n$.

$A = \left(\frac{1}{4}\right)^n$ or $A = \frac{1}{4^n}$ or $A = 4^{-n}$

6. Graph both of your functions from Question 5. Do they produce the same graph as in Question 4?

YES!

7. What will be the exact area of the smallest triangle if the pattern is continued to the sixth term?

$A = \frac{1}{4^6}$ or $\frac{1}{4,096}$

8. As n gets larger and larger, what happens to A?

A gets closer and closer to ZERO.

9. According to your function, can the area of the smallest triangle ever be 0?

No, but it can be very very small and can get very close to ZERO.

10. The function you found in this Investigation is an example of **exponential decay**. Summarize the properties of this function.

The function $A = \left(\frac{1}{4}\right)^n$ decreases as the input values increase (it is exponential, not linear). This function crosses the vertical axis (A-axis) at 1 (the initial value of the area), but never crosses the horizontal axis (n-axis). The domain of this function includes all whole numbers and the range includes all positive numbers that are less than or equal to 1.