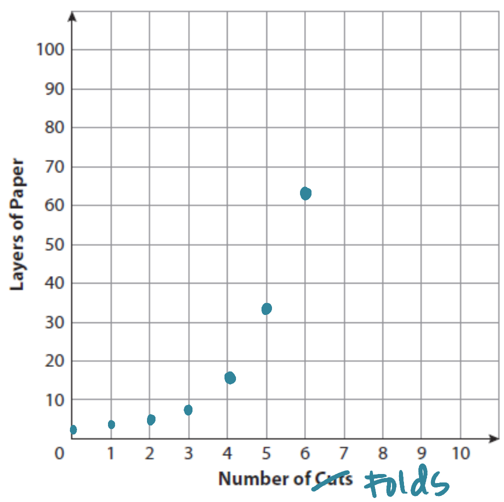
10.1 Activity

1. ~~Cut~~ Fold a piece of paper in half. Lay the half-sheets on top of each other. Count and record the number of sheets in the pile. ~~Cut each~~ Fold of the two half-sheets in half again. Count and record. Continue stacking and ~~cutting~~ folding the paper. Each time, count the number of layers of paper in the stack and record the result in a table like the one shown below.

Number of ~~Cuts~~ Folds	Number of Layers of Paper			
0	1			
1	2			
2	4			
3	8			
4	16			
5	32			
6	64			

2. Make a scatter plot of your data on a grid similar to the one below.



3. Is the function you have graphed continuous or discrete? Explain.

Discrete – not including portion of a fold (whole #'s)

4. What are the domain and range of the function? D: {0, 1, 2, 3, 4, 5, 6, ...} R: {1, 2, 4, 8, 16, 32, 64, ...}

whole #'s

5. Label the third column of your table "Change in Number of Layers." For each cut, write the difference between the number of layers after the cut and before the cut.

Number of Cuts	Number of Layers of Paper	Change in Number of Layers		
0	1	—		
1	2	2 − 1 = 1		
2	4	4 − 2 = 2		
3	8	8 − 4 = 4		
4	16	16 − 8 = 8		
5	32	32 − 16 = 16		
6	64	64 − 32 = 32		

6. Describe the pattern of the changes in the number of layers. The change doubles w/ each fold.

7. Is the change in the number of layers a linear function of the number of cuts? Explain. No, the rate of change is not constant.

8. Label the fourth column of your table "Ratio of Successive Numbers of Layers." For each cut, write the ratio of the number of layers after the cut to the number of layers before the cut. For example, after the first cut,

$$\frac{\text{number of layers after 1 cut}}{\text{number of layers after 0 cuts}} = \frac{2}{1} = 2$$

Describe any pattern you see in the fourth column. The ratio is constant

Number of Cuts	Number of Layers of Paper	Change in Number of Layers	Ratio of Successive Numbers of Layers	
0	1	—	—	
1	2	$2 - 1 = 1$	$\frac{2}{1} = 2$	
2	4	$4-2=2$	$\frac{4}{2}=2$	
3	8	4	$\frac{8}{4}=2$	
4	16	8	$\frac{16}{8}=2$	
5	32	16	$\frac{32}{16}=2$	
6	64	32	$\frac{64}{32}=2$	

9. Label the fifth column of your table "Computing Process for Column 2." In this column, show how powers of 2 can be used to compute the number of layers of paper after each cut. For example, after the second cut, the number of layers is $2 \cdot 2 = 2^2$.

Number of Cuts	Number of Layers of Paper	Change in Number of Layers	Ratio of Successive Numbers of Layers	Computing Process for Column 2
0	1	—	—	$2^0=1$
1	2	$2 - 1 = 1$	$\frac{2}{1} = 2$	$2 = 2^1$
2	4	$4 - 2 = 2$	$\frac{4}{2} = 2$	$2 \cdot 2 = 2^2$
3	8	4	2	$2 \cdot 2 \cdot 2 = 2^3$
4	16	8	2	$2 \cdot 2 \cdot 2 \cdot 2 = 2^4$
5	32	16	2	2^5
6	64	32	2	2^6

10. Look at the first and fifth columns of your table. Describe any connection you see between the number of cuts and the expressions in the "Computing Process for Column 2" column.

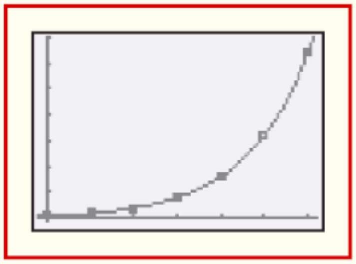
The exponent is equal to the # of folds.

11. Write a function for the number of layers of paper L if you cut the paper n times.

$L(n) = 2^n$

12. Use a graphing calculator to make a scatter plot of the data from your first table in Question 1. Then graph your function from Question 11 on the scatter plot and record your window. Does your function describe the points in the scatter plot?

Yes; The graph of the function passes through all of the points.



[−0.2, 6.2] × [−1, 70]

13. Use your function to predict how many layers of paper there would be after 10 cuts. $L(10) = 2^{10} = 1{,}024$ layers

14. What happens as the number of cuts increases? The # of layers continues to double w/o limit. (although if we fold... at some point we won't be able to fold anymore...)

15. What is the vertical intercept of the graph of your function? 1

16. Write a summary of what you have found out about the properties of the function that relates the number of layers of paper to the number of cuts.

The function is not linear, because there is not a constant rate of change. However, there is a constant ratio found by using successive quotients. This is a discrete situation with a domain of whole #'s and a range of $y = 2^n$ where n is a whole #.