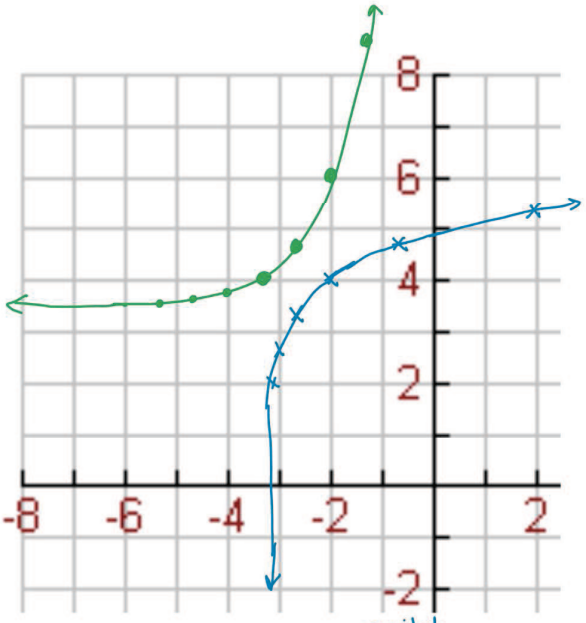
Discovering the Logarithmic Function

Name Key ☺

No Calculator

1. Graph $f(x) = 2^x$.



2. Now, find the inverse of $f(x) = 2^x$.
   - Graphically
   - Algebraically

switch coordinates

$f(x) = 2^x$

$y = 2^x \rightarrow$ inverse $\rightarrow x = 2^y$

power, exponent, base

· switch x & y
· solve for y

power, base

$\log_2 x = y$ ← exponent

base

Read: "log base 2 of $x = y$"

3. Complete the table by using the graphs above:

	$f(x) = 2^x$	$f^{-1}(x) =$ $\log_2 x$
Domain	$\mathbb{R}$	$x > 0$
Range	$y > 0$	$\mathbb{R}$
Equation of Asymptote	$y = 0$	$x = 0$

Generalize the relationship between exponentials and logarithms:

- Exponentials are inverses of logarithms and vice versa.
- WEGO = "What exponent goes on 3 to get 81?"

Rewrite in logarithmic form:

4. $3^4 = 81$ (exponent, power, base) $\log_3 81 = 4$

5. $81^{\frac{1}{2}} = 9$ $\log_{81}(9) = \frac{1}{2}$

6. $16^{\frac{-5}{4}} = \frac{1}{32}$ $\log_{16}\left(\frac{1}{32}\right) = -\frac{5}{4}$

Rewrite in exponential form:

7. $\log_2 128 = 7$ $2^7 = 128$

8. $\log_9 \frac{1}{3} = \frac{-1}{2}$ $9^{-1/2} = \frac{1}{3}$ $\frac{1}{9^{1/2}} = \frac{1}{3}$ $\frac{1}{\sqrt{9}} = \frac{1}{3}$ ✓

9. $\log_8 \frac{1}{4} = \frac{-2}{3}$ $8^{\frac{-2}{3}} = \frac{1}{4}$ $\frac{1}{8^{2/3}} = \frac{1}{4}$ $\frac{1}{(8^{1/3})^2} = \frac{1}{4}$ ✓ $\frac{1}{2^2}$

Evaluate the following logarithms:

10. $\log_4 16 = x$ $4^x = 16$ $x = 2$

11. $\log_6 216 = x$ $6^x = 216$ $x = 3$

12. $\log_2 4 = x$ $2^x = 4$ $x = 2$

13. $\log_3 27 = x$ $3^x = 27$ $x = 3$

14. $\log_{10} 1000 = x$ $10^x = 1000$ $x = 3$

15. $\log_2 16 = x$ $2^x = 16$ $x = 4$

16. $\log_5 125 = x$ $5^x = 125$ $x = 3$

17. $\log_4 \frac{1}{4} = x$ $4^x = \frac{1}{4}$ $x = -1$

18. $\log_6 1 = x$ $6^x = 1$ $x = 0$

19. $\log_2 \frac{1}{4} = x$ $2^x = \frac{1}{4}$ $x = -2$

20. $\log_{10} \frac{1}{100} = x$ $10^x = \frac{1}{100}$ $x = -2$

21. $\log_2 \frac{1}{16} = x$ $2^x = \frac{1}{16}$ $x = -4$

22. $\log_2 \frac{1}{32} = x$ $2^x = \frac{1}{32}$ $x = -5$

23. $\log_8 32 = x$ $8^x = 32$ $(2^3)^x = 2^5$ $3x = 5$ $x = \frac{5}{3}$

24. $\log_4 \frac{1}{8} = x$ $4^x = \frac{1}{8}$ $(2^2)^x = 2^{-3}$ $2x = -3$ $x = -\frac{3}{2}$

25. $\log_{16} \frac{1}{64} = x$ $16^x = \frac{1}{64}$ $(4^2)^x = 4^{-3}$ $2x = -3$ $x = -\frac{3}{2}$

26. $\log_{100} .001 = x$ $100^x = .001$ $100^x = 10^{-4}$ $(10^2)^x = 10^{-4}$ $x = -2$