11.1 HW Key

1 **a.** The directrix of the parabola in the first part of the Investigation was below the focus. What happens as you move the focus closer to the directrix? (If you are not sure, use focus-directrix paper and try placing the directrix 1 unit away from the focus, rather than 2 units. What happens to the parabola?)

a) The "U" becomes more narrow

b. What happens if you move the directrix farther away from the focus? (For example, place the directrix 4 units away from the focus, rather than 1 or 2 units.)

b) The "U" becomes wider.

c. How can you create a parabola so that it opens to the right? (Hint: try drawing a graph.)

c) Draw a vertical directrix and put the focus to the right of it.

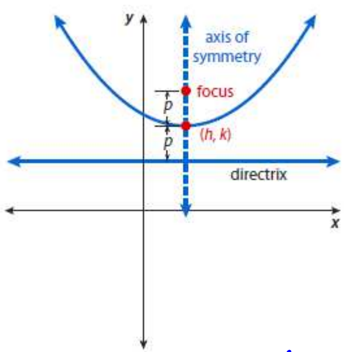
d. How can you create a parabola so that it opens to the left?

e. Do the graphs in Parts (c) and (d) represent functions? Explain.

d) Draw a vertical directrix and put the focus to the left of it.

e) No, they do not pass the Vertical Line Test (more than one output for an input)

2. Think about a parabola with its vertex at the origin and the y-axis as its axis of symmetry. If the parabola is shifted h units horizontally and k units vertically, then the result is a parabola with its vertex at (h, k) and an axis of symmetry parallel to the y-axis. The standard form of an equation for this parabola is $(x - h)^2 = 4p(y - k)$.

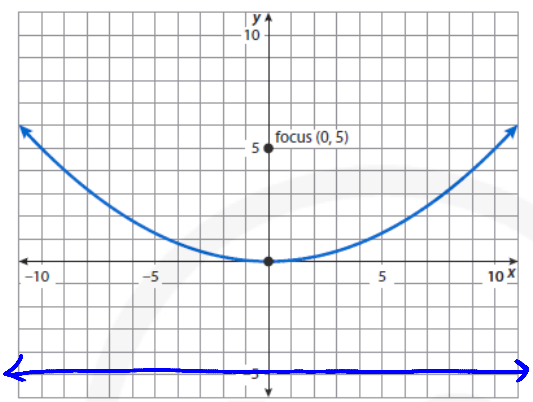


a. What are the coordinates of the focus? $(h, k+p)$

b. What is the equation of the axis of symmetry? $x = h$

c. What is the equation of the directrix? $y = -p + k$

3. Find equations of the directrix and the parabola.



Parabola:

Vertex $(0,0)$

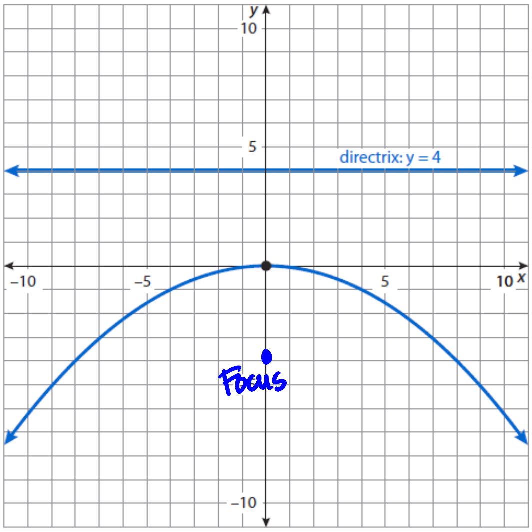
$p = 5 \therefore x^2 = 4py$

$x^2 = 4(5)y$

$\boxed{x^2 = 20y}$

Directrix: $\boxed{y = -5}$

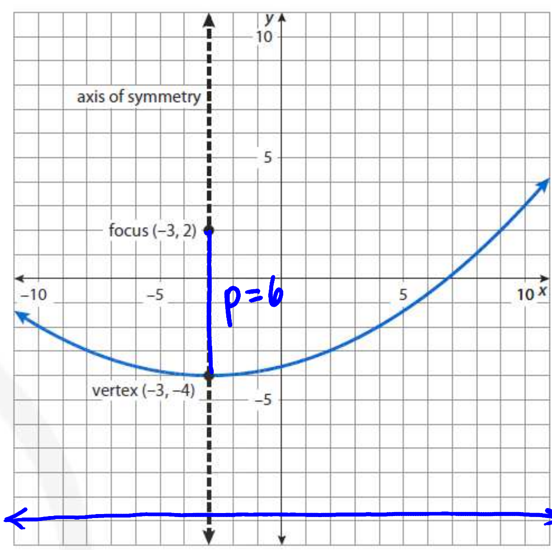
4. Find the focus and an equation of the parabola.



$\boxed{\text{Focus } (0,-4)}$

Parabola:
Vertex $(0,0)$
$p=-4 \therefore\ x^2=4py$
$x^2=4(-4)y$
$\boxed{x^2=-16y}$

5. Find an equation of the directrix and an equation of the parabola. ★Be careful... Vertex is @ $(-3,-4)$★ (not @ the origin)



Directrix: $\boxed{y=-10}$

Parabola
$p=6 \therefore\ (x-h)^2=4p(y-k)^2$
$(x--3)^2=4(6)(y--4)^2$
$(x+3)^2=24(y+4)^2$

For Exercises 6–9, find the vertex, the focus, and the equation of the directrix for each parabola. Then draw the graph.

6. $x^2 = 8y$ Vertex: $(0,0)$ Focus: $(0,2)$ Directrix: $y=-2$

7. $x^2 = -16y$ Vertex: $(0,0)$ Focus: $(0,-4)$ Directrix: $y=4$

8. $(x-1)^2 = -4(y+3)$

9. $(x+2)^2 = 2(y-5)$

6. $\boxed{\text{Vertex}: (0,0)}$
$\boxed{\text{Focus}: (0,2)}$ ← $(h,k+p)$
$x^2=4py$
$x^2=8y$

7. $\boxed{\text{Vertex}: (0,0)}$
$\boxed{\text{Focus}: (0,-4)}$ ← $(h,k+p)$
$x^2=4py$

$x^2 = 4py$

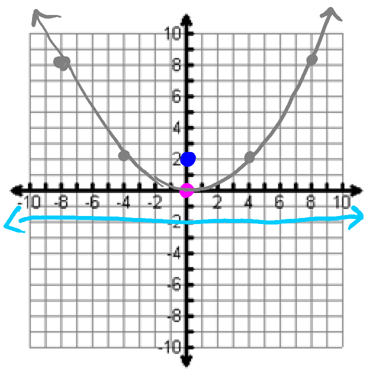
$x^2 = 8y$

$4py = 8y$

$p = 2$

$\boxed{\text{Directrix: } y = -2}$

$y = -p + k$



x	y
0	0
4	2
-4	2
8	8
-8	8

$x^2 = 4py$

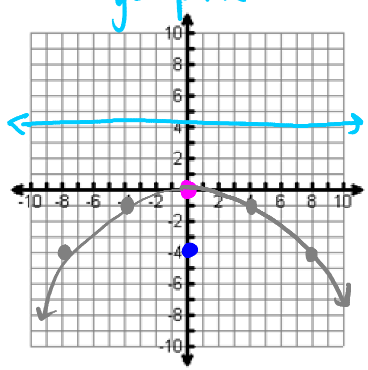
$x^2 = -16y$

$4py = -16y$

$p = -4$

$\boxed{\text{Directrix: } y = 4}$

$y = -p + k$



x	y
0	0
4	-1
-4	-1
8	-4
-8	-4

8. $(x-1)^2 = -4(y+3)$

$\boxed{\text{Vertex: } (1,-3)}$

Focus: $(h, k+p) \rightarrow (1, -3+-1)$

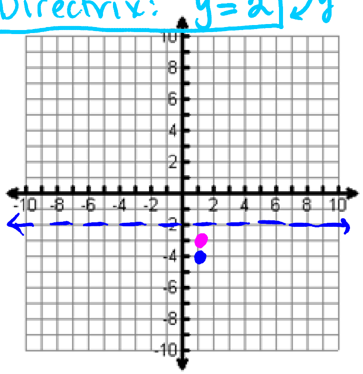
$x^2 = 4py$ $\boxed{\text{Focus: } (1,-4)}$

$x^2 = -4y$

$4py = -4y$

$p = -1$

$\boxed{\text{Directrix: } y = -2}$ $\swarrow y = -p + k$



x	y
1	-3

9. $(x+2)^2 = 2(y-5)$

$\boxed{\text{Vertex: } (-2,5)}$

Focus: $(h, k+p) \rightarrow (-2, 5 + \frac{1}{2})$

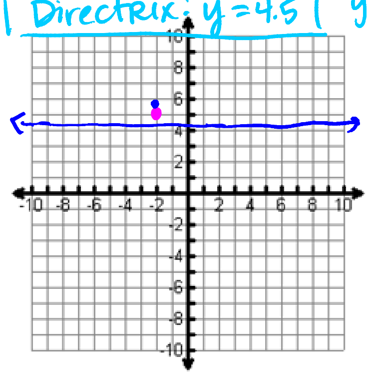
$x^2 = 4py$ $\boxed{\text{Focus: } (-2, 5.5)}$

$x^2 = 2y$

$4py = 2y$

$p = 1/2$

$\boxed{\text{Directrix: } y = 4.5}$ $y = -p + k$



10. State whether the following statements are true or false. Then explain your reasoning.

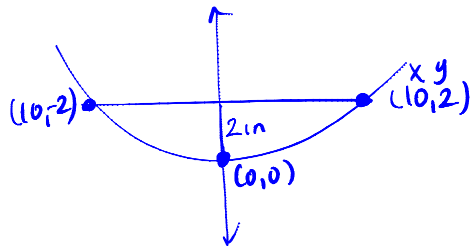
Statement A: All parabolas are graphs of functions.

Statement B: All equations of the form $x^2 = 4py$ are functions and can be represented by parabolas.

A) False → some parabolas can open to the left/right and these are not functions

B) True → The graphs of $x^2 = 4py$ are of parabolas that open up or down.

11. A satellite dish has a diameter of 20 inches and a depth of 2 inches. Where should the receiver be located? Explain.



$x^2 = 4py$

$10^2 = 4p(2)$

$100 = 8p$

$\frac{100}{8} = p$

$\frac{25}{2} = p$

$12.5 = p$

The receiver should be placed @ the focus which is outside the satellite dish - 12.5 in from the vertex of the parabola.