COMMON CORE

1-4 Pairs of Angles

CC.9-12.G.CO.1 Know precise definitions…based on the undefined notions of point, line,…

Objectives
Identify adjacent, vertical, complementary, and supplementary angles.

Find measures of pairs of angles.

Vocabulary
adjacent angles
linear pair
complementary angles
supplementary angles
vertical angles

Who uses this?

Scientists use properties of angle pairs to design fiber-optic cables. (See Example 4.)

A fiber-optic cable is a strand of glass as thin as a human hair. Data can be transmitted over long distances by bouncing light off the inner walls of the cable.

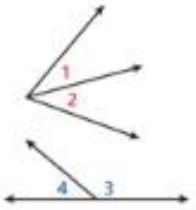
Many pairs of angles have special relationships. Some relationships are because of the measurements of the angles in the pair. Other relationships are because of the positions of the angles in the pair.

Know it! Note

Pairs of Angles

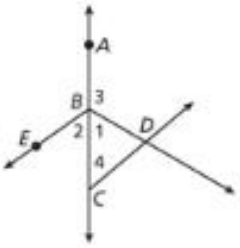
Adjacent angles are two angles in the same plane with a common vertex and a common side, but no common interior points. $\angle 1$ and $\angle 2$ are adjacent angles.

A **linear pair** of angles is a pair of adjacent angles whose noncommon sides are opposite rays. $\angle 3$ and $\angle 4$ form a linear pair.



EXAMPLE 1 Identifying Angle Pairs

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.



A $\angle 1$ and $\angle 2$

$\angle 1$ and $\angle 2$ have a common vertex, B, a common side, $\overrightarrow{BC}$, and no common interior points. Therefore $\angle 1$ and $\angle 2$ are only adjacent angles.

B $\angle 2$ and $\angle 4$

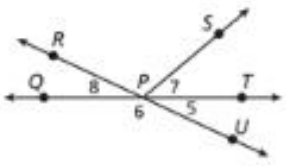
$\angle 2$ and $\angle 4$ share $\overline{BC}$ but do not have a common vertex, so $\angle 2$ and $\angle 4$ are not adjacent angles.

C $\angle 1$ and $\angle 3$

$\angle 1$ and $\angle 3$ are adjacent angles. Their noncommon sides, $\overrightarrow{BC}$ and $\overrightarrow{BA}$, are opposite rays, so $\angle 1$ and $\angle 3$ also form a linear pair.

CHECK IT OUT!

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.



1a. $\angle 5$ and $\angle 6$

1b. $\angle 7$ and $\angle SPU$

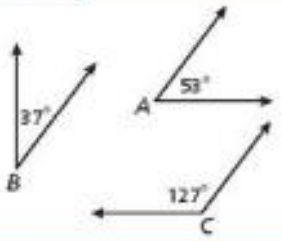
1c. $\angle 7$ and $\angle 8$

Know it! Note

Complementary and Supplementary Angles

Complementary angles are two angles whose measures have a sum of 90°. $\angle A$ and $\angle B$ are complementary.

Supplementary angles are two angles whose measures have a sum of 180°. $\angle A$ and $\angle C$ are supplementary.



You can find the complement of an angle that measures $x°$ by subtracting its measure from 90°, or $(90 - x)°$. You can find the supplement of an angle that measures $x°$ by subtracting its measure from 180°, or $(180 - x)°$.

EXAMPLE 2 Finding the Measures of Complements and Supplements

Find the measure of each of the following.

A complement of $\angle M$

$(90 - x)°$

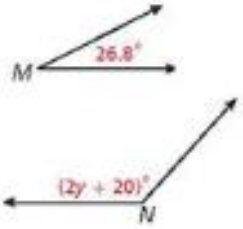
$90° - 26.8° = 63.2°$

B supplement of $\angle N$

$(180 - x)°$

$180° - (2y + 20)° = 180° - 2y - 20$

$= (160 - 2y)°$

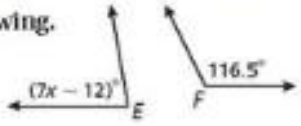


CHECK IT OUT!

Find the measure of each of the following.

2a. complement of $\angle E$

2b. supplement of $\angle F$



EXAMPLE 3 Using Complements and Supplements to Solve Problems

Algebra

An angle measures 3 degrees less than twice the measure of its complement. Find the measure of its complement.

Step 1 Let $m\angle A = x°$. Then $\angle B$, its complement, measures $(90 - x)°$.

Step 2 Write and solve an equation.

$m\angle A = 2m\angle B - 3$

$x = 2(90 - x) - 3$ *Substitute x for m∠A and 90 − x for m∠B.*

$x = 180 - 2x - 3$ *Distrib. Prop.*

$x = 177 - 2x$ *Combine like terms.*

$+2x \quad +2x$ *Add 2x to both sides.*

$3x = 177$ *Simplify.*

$\frac{3x}{3} = \frac{177}{3}$ *Divide both sides by 3.*

$x = 59$ *Simplify.*

The measure of the complement, $\angle B$, is $(90 - 59)° = 31°$.

CHECK IT OUT!

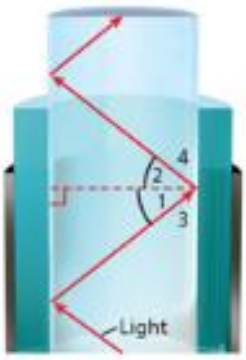
3. An angle's measure is 12° more than $\frac{1}{2}$ the measure of its supplement. Find the measure of the angle.

EXAMPLE 4 Problem-Solving Application

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

Light passing through a fiber optic cable reflects off the walls in such a way that $\angle 1 \cong \angle 2$. $\angle 1$ and $\angle 3$ are complementary, and $\angle 2$ and $\angle 4$ are complementary.
If $m\angle 1 = 38°$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.



1 Understand the Problem

The answers are the measures of $\angle 2$, $\angle 3$, and $\angle 4$.
List the important information:

- $\angle 1 \cong \angle 2$
- $\angle 1$ and $\angle 3$ are complementary, and $\angle 2$ and $\angle 4$ are complementary.
- $m\angle 1 = 38°$

2 Make a Plan

If $\angle 1 \cong \angle 2$, then $m\angle 1 = m\angle 2$.
If $\angle 3$ and $\angle 1$ are complementary, then $m\angle 3 = (90 - 38)°$.
If $\angle 4$ and $\angle 2$ are complementary, then $m\angle 4 = (90 - 38)°$.

3 Solve

By the Transitive Property of Equality, if $m\angle 1 = 38°$ and $m\angle 1 = m\angle 2$, then $m\angle 2 = 38°$. Since $\angle 3$ and $\angle 1$ are complementary, $m\angle 3 = 52°$. Similarly, since $\angle 2$ and $\angle 4$ are complementary, $m\angle 4 = 52°$.

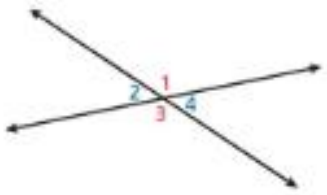
4 Look Back

The answer makes sense because $38° + 52° = 90°$, so $\angle 1$ and $\angle 3$ are complementary, and $\angle 2$ and $\angle 4$ are complementary. Thus $m\angle 2 = 38°$, $m\angle 3 = 52°$, and $m\angle 4 = 52°$.

CHECK IT OUT!

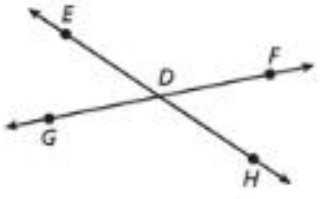
4. What if...? Suppose $m\angle 3 = 27.6°$. Find $m\angle 1$, $m\angle 2$, and $m\angle 4$.

Another angle pair relationship exists between two angles whose sides form two pairs of opposite rays. **Vertical angles** are two nonadjacent angles formed by two intersecting lines. $\angle 1$ and $\angle 3$ are vertical angles, as are $\angle 2$ and $\angle 4$.



EXAMPLE 5 Identifying Vertical Angles

Name one pair of vertical angles.
Do they appear to have the same measure?
Check by measuring with a protractor.



$\angle EDF$ and $\angle GDH$ are vertical angles and appear to have the same measure.

Check $m\angle EDF \approx m\angle GDH \approx 135°$.

CHECK IT OUT!

5. Name another pair of vertical angles. Do they appear to have the same measure? Check by measuring with a protractor.